«Сейфуллин оқулары - 11: Жастар және ғылым» атты Республикалық ғылыми-теориялық конференциясының материалдары = Материалы Республиканской научно-теоретической конференции **«Сейфуллинские чтения - 11: Молодежь и наука».** – 2015. – **Т.1, ч.3**. – С. 206-209

ABOUTMETHODFORAPPROXIMATESOLUTION OF LINEAR BOUNDARY VALUE PROBLEM

Let $\Omega = \Delta ABC$istriangle with sides$\partial\Omega = [AB] \cup [BC] \cup [AC]$, where

$$AB : x = \frac{l}{4}, y \in \left[\frac{l}{4}, \frac{l}{2}\right],\ BC : y = \frac{3l}{4} - x, x \in \left[\frac{l}{4}, \frac{l}{2}\right],\ AC : y = \frac{l}{4}, x \in \left[\frac{l}{4}, \frac{l}{2}\right].$$

Suppose that

$$-\Delta u + 5xu = -f(x, y),\ f(x, y) \in L_2(\Omega)$$

with the boundary condition

$$u\big|_{\partial\Omega} = 0.$$

Let Q = [0,l] x [0,l] be square and

$$-\Delta u + u = v$$
$$u\big|_{x=0} = u\big|_{x=l} = u\big|_{y=l} = 0$$

is elliptic problem with periodic boundary conditions.

The solution of this problem can be written as

$$u = Av = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{\left(\frac{\pi n}{l}\right)^2 + \left(\frac{\pi m}{l}\right)^2} \sin\frac{\pi nx}{l} \sin\frac{\pi my}{l} \int_0^l d\xi \int_0^l v(\xi, \eta) \sin\frac{\pi n\xi}{l} \sin\frac{\pi n\xi}{l} d\eta.$$

In this case we can represent the original problem in the form as [2]

$$v + (5x-1)Av = f(x,y).$$

The left side of the equation is defined by

$$Mv = v + (5x - 1)Av.$$

Then the equation can be written as follows

$$Mv = f(x, y).$$

We determine the functional:

$$J = \int_0^l dx \int_0^l \chi(x)|Mv - f|^2 dx + \int_{AB} |AB|^2 dy + \int_{BC} |Av|^2 dxdu + \int_{AC} |Av|^2 dx.$$

$$\chi(x) = \begin{cases} 1, & x \in \Omega \\ 0, & x \in Q / \Omega \end{cases}.$$

Operator Av at the boundary AB can be written as follows [4]:

$$Av_1 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\left(\frac{\pi n}{l}\right)^2 + \left(\frac{\pi m}{l}\right)^2} \sin \frac{\pi n x}{4} \sin \frac{\pi m y}{l} \int_0^l d\xi \int_0^l v(\xi, \eta) \sin \frac{\pi n \xi}{l} \sin \frac{\pi m \eta}{l} d\eta,$$

at the border*BC:*

$$Av_2 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\left(\frac{\pi n}{l}\right)^2 + \left(\frac{\pi m}{l}\right)^2} \sin \frac{\pi n x}{4} \sin \frac{\pi m y}{l} \int_0^l d\xi \int_0^l v(\xi, \eta) \sin \frac{\pi n \xi}{l} \sin \frac{\pi m \eta}{l} d\eta,$$

at the border*AC:*

$$Av_3 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\left(\frac{\pi n}{l}\right)^2 + \left(\frac{\pi m}{l}\right)^2} \sin \frac{\pi n x}{l} \sin \frac{\pi m y}{4} \int_0^l d\xi \int_0^l v(\xi, \eta) \sin \frac{\pi n \xi}{l} \sin \frac{\pi m \eta}{l} d\eta.$$

Therefore, the functional has the form:

$$J = \int_0^l dx \int_0^l |Mv - f|^2 dy + \langle N_1 v_1, v_1 \rangle + \langle N_2 v_2, v_2 \rangle + \langle N_3 v_3, v_3 \rangle,$$

where$N_1 v_1 = A^* A v_1$, $N_2 v_2 = A^* A v_2$, $N_3 v_3 = A^* A v_3$

$$N_1 v_1 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\left(\frac{\pi n}{l}\right)^2 + \left(\frac{\pi m}{l}\right)^2} \sin \frac{\pi n \xi}{l} \sin \frac{\pi m \eta}{4} \int_{\frac{l}{4}}^{\frac{l}{2}} A v_1(y) \sin \frac{\pi n}{4} \sin \frac{\pi m y}{l} dy,$$

$$N_2 v_2 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\left(\frac{\pi n}{l}\right)^2 + \left(\frac{\pi m}{l}\right)^2} \cdot \sin \frac{\pi n \xi}{l} \sin \frac{\pi m \eta}{4} \int_{\frac{l}{4}}^{\frac{l}{2}} dx \int_{\frac{l}{4}}^{\frac{3l}{4} - x} A v_2(x, y) \sin \frac{\pi n x}{4} \sin \frac{\pi m y}{l} dy,$$

$$N_3 v_3 = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{\left(\frac{\pi n}{l}\right)^2 + \left(\frac{\pi m}{l}\right)^2} \cdot \sin\frac{\pi n\xi}{l} \sin\frac{\pi m\eta}{4} \int_{\frac{l}{4}}^{\frac{l}{2}} A v_3(x) \sin\frac{\pi n x}{l} \sin\frac{\pi m}{4} dx.$$

We introduce the scheme the approximate solution of the problem [1]:

$$J = \int_0^l dx \int_0^l \left| M\omega_n - f \right|^2 dy + \left\langle N_1\omega_1, \omega_1 \right\rangle + \left\langle N_2\omega_n, \omega_n \right\rangle + \left\langle N_3\omega_n, \omega_n \right\rangle,$$

$$\omega_{n+1} = \varpi_n + \varepsilon[M^*(M(\omega_n - f) + N_1\varpi_n + N_2\varpi_n + N_3\omega_n] = \varpi_n + \varepsilon[(M\varpi_n - f) +$$
$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{\left(\frac{\pi n}{l}\right)^2 + \left(\frac{\pi m}{l}\right)^2} \cdot \sin\frac{\pi n x}{l} \sin\frac{\pi m y}{4} \int_0^l d\xi \int_0^l (M\omega_n - f) \sin\frac{\pi n\xi}{4} \sin\frac{\pi m\eta}{l} d\eta +$$
$$+ N_1\omega_n + N_2\omega_n + N_3\varpi_n].$$

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