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SOLUTION OF APPLIED PROBLEMS USING DIFFERENTIAL EQUATIONS

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A modern specialist, a university graduate should know not only his specialty, but also have the skill of research, apply the knowledge gained in solving specific problems. The study of scientific and mathematical disciplines plays a huge role in the development of these qualities. One of the means of forming professional competence of future engineers is teaching students to solve applied problems in mathematics.

For students of technical specialties, one of the most important sections of mathematics is "Differential Equations". With the conclusion and application of differential equations to the solution of certain applied problems students are faced with the study of various general or special courses, such as physics, theoretical mechanics, resistance of materials, theory of machines and mechanisms, electrical engineering, etc.[1]

A necessary step in solving any applied problem is the construction of a mathematical model of the studied object or process. Ordinary differential equations of the first and second order form the basis of simple, but very common mathematical models used in various fields of science and technology. However, making a mathematical model of the process under consideration requires knowledge both on the topic of "Differential equations" and in a specific applied field. Practice shows that at the solution of applied problems students have difficulties connected with the formulation of the differential equation of the process, presented in the condition of the problem. This is due to the fact that the textbooks do not specify the order of actions, the execution of which is necessary for the development of the differential equation of the process under consideration.

Many researches are devoted to the theme of teaching the solution of applied problems of differential equations. In works of N.V.Poleukhovich schemes of the solution of applied physical and geometrical problems are presented. The scheme for solving applied physical problems consists of three stages: compiling of differential equation, work with differential equation and interpretation of results. [2]

To compose a differential equation it is necessary:

- 1) to determine the physical law regulating the process considered in the condition of the problem;
- 2) to write down the equality corresponding to this law;
- 3) to determine which of the values is an independent variable;

4) to express the changing values through the independent variable and the problem data.

In the case when the law is unknown, it is necessary:

1) select quantities, one of which will be an independent variable, the other with the desired function, introduce notation for them;

2) express the change in the desired function, which will correspond to the increment of the independent variable.

To solve applied geometrical problems first of all it is necessary to make the drawing and to put on it all given problems. Then the condition set in the task should be written down as equality. If a formula is indicated in the problem for each component of this equality, then applying it, draw up a differential equation based on the equation specified in the condition.

If there is no such formula, then:

1) mark the coordinates of an arbitrary curve point through $(x; y)$ and indicate in the drawing the angle α of the tangent passing through this point;

2) express the required values through the variables, x, y, y', y'', \dots , where $y' = \operatorname{tg} \alpha$;

3) to compose a differential equation on the basis of the one given in the condition of equality [3].

We give examples of problems in the preparation of differential equations.

Problem 1: The material point moves on a straight line with constant acceleration a . Find the law of the point movement. [5]

The acceleration a represents the derivative of speed v with respect to time t , or the second drive-by path from s with respect to time t , i.e.

$$s'' = a, \text{ or } \frac{dv}{dt} = a, \text{ so } dv = a dt.$$

We got a differential equation representing a mathematical model of a physical process whose solution is the function

$$s = \frac{1}{2} at^2 + v_0 t + s_0.$$

Problem 2. The curve passing through points $A(5;7)$ and $B(6;6)$ has a radius of curvature $R=5$. Find equation of this curve.

The radius of curvature is determined by the formula

$$R = \frac{[1 + y'^2]^{3/2}}{y''}.$$

Using the condition of problem $R=5$, we get the differential equation

$$\frac{[1 + y'^2]^{3/2}}{y''} = 5$$

The general solution to this equation is as follows:

$$(x - C_1)^2 + (y - C_2)^2 = 25.$$

Considering that the curve passes through points $A(5;7)$ and $B(6;6)$, we find the equation of the curve.

$$(x - 2)^2 + (y - 3)^2 = 25.$$

The given examples show that the ordinary differential equations are the widespread mathematical models applied for the solution of problems in the most different fields of science and technology.

The derivation of differential equations is an important but also difficult issue. A universal method that is suitable in all cases cannot be specified. It is necessary to acquire experience and certain skills in solving problems, which is achieved by analyzing a large number of problems and solving similar examples independently [5]. After obtaining a solution of an applied problem, it is important to be able to comprehend and analyze the obtained result, give it a practical interpretation and try to draw useful conclusions aimed at improving the object or process in question.

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